

A Complex Integral Transform “Complex *EE* Transform” and Its Applications

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Abstract

In this paper, a new complex integral transform under the name EE (Emad-Elaf) transform had been suggested and applied to solve linear ordinary differential equations with constant coefficient, the new EE transform had been tested by applying it on some practical problems, and it proved to be an efficient and powerful transform.

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1. Introduction

The new EE transform, that proposed by Emad A. Kuffi and Elaf Sabah Abbas has been driven from Fourier and Laplace integrals. Due to the importance of differential equation in scientific and engineering fields they became a target for the researchers, many methods had been proposed to solve differential equations, such as Fourier, Laplace, Elzaki, Abood, Mohand, Al-Zughair, Kamal, Mahgoub, SEE and complex SEE transforms, [1-10].

The new proposed EE transform and its fundamental properties provided a mathematical simplicity that made it very suitable to facilitate the solution of differential equations, and it had been used to solve ordinary differential equations in the time domain.

The new integral transform *EE* transform is defined for a function of exponential order, we consider the function $f(t)$ in the set A is defined by:

$$A = \{f(t): \exists M, l_1, l_2 > 0, |f(t)| < Me^{l_j|t|}, \text{ if } (-1)^j \times [0, \infty)\} \quad (1)$$

For the given function in the set of A , the constant M must be finite number $l_j (j = 1, 2)$ may be finite or infinite.

EE integral transform denoted by the operator $E\{\cdot\}$ defined by the integral equation:

$$E^c\{f(t)\} = E(iv) = \int_{t=0}^{\infty} f(t)e^{-iv^n t} dt \quad , n \in \mathbb{Z}, t \geq 0 .$$

$v \in [l_1, l_2], i$ complex number.

The variable v in this integral is used to factor the variable t in the argument of the function f . This integral transform has deeper connection with the Fourier, Laplace and complex SEE transform.

The purpose of this study is to show the applicability of this interesting new integral transform and its efficiency in solving the linear ordinary differential equations with constant coefficients.

2. EE Complex Integral Transform of some Functions

For any function $f(t)$, assuming that the integral in equation (2) exists. The sufficient conditions for the existence of EE complex integral transform is that $f(t), t \geq 0$ be a piecewise continuous and of exponential order, otherwise EE complex integral transform may or may not exists.

In this section, the EE complex integral transform is found for simple functions:

- (a) If $f(t) = C$, where C is a constant function, then by the definition,

$$E^c\{C\} = \int_{t=0}^{\infty} C e^{-iv^n t} dt = \frac{-C}{iv^n} [0 - 1] = \frac{-Ci}{v^n} \Rightarrow E^c\{C\} = \frac{-Ci}{v^n}.$$
- (b) If $f(t) = t$, then $E^c\{t\} = \int_{t=0}^{\infty} t e^{-iv^n t} dt$ (Integration by parts): $E^c\{t\} = \frac{-1}{v^{2n}}.$
- (c) $E^c\{t^2\} = \frac{2!i}{v^{3n}}.$
- (d) $E^c\{t^3\} = \frac{3!i}{v^{4n}}.$
- (e) In general, $E^c\{t^m\} = \frac{m!}{(iv)^{(m+1)n}}, m$ is a positive integral number.
- (f) If $f(t) = e^{at}$ where a is a constant number, then: $E^c\{e^{at}\} = \int_{t=0}^{\infty} e^{at} e^{-iv^n t} dt = \int_{t=0}^{\infty} e^{-(iv^n - a)t} dt \Rightarrow$

$$E^c\{e^{at}\} = -\left[\frac{a}{a^2 + v^{2n}} + i \frac{v^n}{a^2 + v^{2n}}\right].$$
- (g) If $f(t) = \sin(at)$, where a is a constant number, then: $E^c\{\sin(at)\} = \frac{1}{2i} \int_{t=0}^{\infty} (e^{iat} - e^{-iat}) e^{-iv^n t} dt,$

$$\Rightarrow E^c\{\sin at\} = \frac{-a}{v^{2n} - a^2}.$$
- (h) If $f(t) = \cos(at)$, where a is a constant number, then: $E^c\{\cos(at)\} = \frac{1}{2} \int_{t=0}^{\infty} (e^{iat} + e^{-iat}) e^{-iv^n t} dt,$

$$\Rightarrow E^c\{\cos(at)\} = \frac{-iv^n}{v^{2n} - a^2}.$$
- (i) If $f(t) = \sinh(at)$, where a is a constant number, then: $E^c\{\sinh(at)\} = \frac{1}{2} \int_{t=0}^{\infty} (e^{at} - e^{-at}) e^{-iv^n t} dt,$

$$\Rightarrow E^c\{\sinh at\} = \frac{-a}{v^{2n} + a^2}.$$
- (j) If $f(t) = \cosh(at)$, where a is a constant number, then: $E^c\{\cosh(at)\} = \frac{1}{2} \int_{t=0}^{\infty} (e^{at} + e^{-at}) e^{-iv^n t} dt,$

$$\Rightarrow E^c\{\cosh(at)\} = \frac{-iv^n}{v^{2n} + a^2}.$$

Theorem 2.1

Let $E(iv)$ is the EE complex integral transform of $[E^c\{f(t)\} = E(iv)]$, then:

- (i) $E^c\{f'(t)\} = -f(0) + iv^n E(iv)$
 (ii) $E^c\{f''(t)\} = -f'(0) - iv^n f(0) - v^{2n} E(iv)$

Proof.

- (i) by the definition: $E^c\{f'(t)\} = \int_{t=0}^{\infty} f'(t) e^{-iv^n t} dt$, Integration by parts: $E^c\{f'(t)\} = -f(0) + iv^n E(iv).$
- (ii) Also, by the definition: $E^c\{f''(t)\} = \int_{t=0}^{\infty} f''(t) e^{-iv^n t} dt$, Integration by parts and use (i): $E^c\{f''(t)\} = -f'(0) - iv^n f(0) - v^{2n} E(iv).$

3. The inverse of EE Complex Integral Transform

If $E^c\{f(t)\} = E(iv)$ is the EE complex integral transform, then $f(t) = EE^{c^{-1}}[E(iv)]$ is called an inverse of EE complex integral transform.

In this section, the inverse of EE transform of simple functions is introduced:

- (a) $E^{c^{-1}}\left\{\frac{-i}{v^n}\right\} = 1.$
 (b) $E^{c^{-1}}\left\{\frac{-1}{v^{2n}}\right\} = t.$
 (c) $E^{c^{-1}}\left\{\frac{2!i}{v^{3n}}\right\} = t^2.$

- (d) $E^{c^{-1}} \left\{ \frac{3!}{v^{4n}} \right\} = t^3$.
- (e) $E^{c^{-1}} \left\{ - \left[\frac{a}{a^2 + v^{2n}} + i \frac{v^n}{a^2 + v^{2n}} \right] \right\} = e^{at}$, a is a constant number.
- (f) $E^{c^{-1}} \left\{ - \frac{a}{v^{2n} - a^2} \right\} = \sin(at)$.
- (g) $E^{c^{-1}} \left\{ - \frac{iv^n}{v^{2n} - a^2} \right\} = \cos(at)$.
- (h) $E^{c^{-1}} \left\{ - \frac{a}{v^{2n} + a^2} \right\} = \sinh(at)$.
- (i) $E^{c^{-1}} \left\{ - \frac{iv^n}{v^{2n} + a^2} \right\} = \cosh(at)$.

4. Applications of EE Complex Integral Transform of Ordinary Differential Equations (ODE_s)

In this section, EE complex integral transform is going to be applied on some practical examples from [8-10], to demonstrate its simplicity in solving integral equation comparing with other transforms.

Example (4.1): consider the differential equation: $\frac{dy}{dx} + 2y = x$, $y(0) = 1$.

Solution:

Taking EE complex integral transform to this equation: $E^c\{y'\} + 2E^c\{y\} = E^c\{x\}$,

$$\text{So, } -y(0) + iv^n E(iv) + 2E(iv) = \frac{-1}{v^{2n}}, (iv^n + 2)E(iv) = \frac{-1}{v^{2n}} + 1, \Rightarrow E(iv) = \frac{-1 + v^{2n}}{v^{2n}(iv^{2n} + 2)}.$$

$$\text{after simple computations: } E(iv) = \frac{i}{4v^n} - \frac{1}{2v^{2n}} - \frac{5}{4} \left[\frac{-2}{4 + v^{2n}} + \frac{iv^n}{4 + v^{2n}} \right].$$

Now, by taking the inverse of both sides of the above equation, the solution is obtained:

$$y(x) = \frac{1}{2}x - \frac{1}{4} + \frac{5}{4}e^{-2x}.$$

Example (4.2): Consider the following second order linear ordinary differential equation: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$, $y(0) = 1$ and $y'(0) = 4$

Solution.

Take EE complex integral transform to above equation:

$$-y'(0) - iv^n y(0) - v^{2n} E(iv) - y(0) + iv^n E(iv) = 0, (-v^{2n} + iv^n)E(iv) = 5 + iv^n.$$

after simple computations:

$$E(iv) = 5 \left(\frac{-i}{v^n} \right) + \frac{4i}{v^n - i}, \Rightarrow E(iv) = 5 \left(\frac{-i}{v^n} \right) + 4 \left(\frac{-1}{v^{2n} + 1} + \frac{iv^n}{v^{2n} + 1} \right)$$

Take the inverse of EE transform of both sides of above equation: $y(x) = 5 - 4e^{-x}$.

Example (4.3): The following problem is based on nuclear physic fundamentals.

Consider the linear first order ordinary differential equation of constant coefficients: $N'(t) = -\gamma N(t)$.

The essential relationship radioactive decay is given above, where $N = N(t)$ during time t denotes the number of un decayed atoms left in a sample of radioactive isotope, and γ is the decay constant. It is possible to use the EE complex integral transform $E^c: E^c \left\{ \frac{dN}{dt} \right\} + \gamma E^c\{N(t)\} = 0$,

$$\text{Therefore: } -N(0) + iv^n E(iv) + \gamma E(iv) = 0$$

$$\text{Here: } E^c\{N(t)\} = E(iv) \text{ and } N(0) = N_0$$

$$\text{So, } E(iv) = \frac{N_0}{\gamma + iv^n}$$

$$\text{Let: } E(iv) = \bar{N}$$

$$\bar{N} = \frac{N_0}{\gamma + iv^n} = \frac{N_0}{\gamma + iv^n} \cdot \frac{\gamma - iv^n}{\gamma - iv^n} = \frac{N_0(\gamma - iv^n)}{\gamma^2 + v^{2n}}, \Rightarrow \bar{N} = -N_0 \left[\frac{\gamma}{\gamma^2 + v^{2n}} + \frac{iv^n}{\gamma^2 + v^{2n}} \right].$$

Now, taking inverse of EE complex integral transform on both sides: $N(t) = N_0 e^{-\gamma t}$.

This is the proper type of radioactive decay.

Example (4.4): Consider the problem of pharmacokinetics: $\frac{d}{dt}C(t) + \lambda C(t) = \frac{\alpha}{v_0 L}$, $t > 0$ with $C(0) = 0$

here $C(t)$ at any time t , the medication concentration in the blood.

λ : is the constant velocity of elimination.

α : is the proportion of the infusion (in mg/min.)

$v_0 L$: volume 0 which drug is distributed.

Now, take EE complex integral transform of both sides of above equation gives: $E^c\{C'(t)\} + \lambda E^c\{C(t)\} = \frac{\alpha}{v_0 L} E^c\{1\}$

Applying EE complex integral transform:

$$-C(0) + iv^n E(iv) + \lambda E(iv) = \frac{\alpha}{v_0 L} \left(\frac{-i}{v^n}\right), \quad (iv^n + \lambda)E(iv) = \frac{-\alpha i}{v_0 L(v^n)}.$$

after simple computation: $c(t) = [1 - e^{-\lambda t}]$.

5. Conclusion.

A new complex integral transform (EE transform) with its fundamental properties had been introduced, and applied on some practical application. The application of the new EE transform had proven the simplicity and capability of this transform in solving differential equations, in which give the EE transform a grate usability in many scientific fields as a powerful integral transform.

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